

# Supersonic Flutter Analysis of Clamped Symmetric Composite Panels Using Shear Deformable Finite Elements

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## Abstract

THE flutter analysis of composite panels in supersonic flow has been performed by the finite element method based on the first-order shear deformable theory. The computational results of the vibration and flutter analysis agree well with the results given in the available references. Flutter boundaries have been obtained for both cross-ply and angle-ply composite plates. Also, the flutter analysis has been performed for both rectangular and trapezoidal plates with clamped edges. The plate aspect ratio, flow direction, and fiber orientation affect greatly the flutter boundaries.

## Contents

Panel flutter is the dynamic aeroelastic instability of plates or shells of the external skin of a flight vehicle exposed to an airflow on one side. Many researchers have studied panel flutter problems for isotropic and orthotropic materials, both theoretically and experimentally.<sup>1</sup> In recent years, for the high performance and weight minimization of the advanced aircraft, the use and the application of composite materials have greatly increased. Sawyer<sup>2</sup> calculated the flutter boundaries for rectangular composite plates by the Galerkin method. Srinivasan and Babu<sup>3</sup> have studied the cross-ply composite panel flutter for both rectangular and trapezoidal plates by the integral equation method. However, the composite panel flutter has not been reported for the trapezoidal plate with angle-ply laminates. In this paper the composite panel flutter phenomena have been analyzed for both rectangular and trapezoidal panels with angle-ply laminates by the finite element method based on the shear deformable theory. The effects of the fiber orientation, geometry shape, and flow direction on flutter boundaries have been studied.

## Finite Element Equations

The composite panel is idealized as a thin plate that has one side exposed to a supersonic airflow and the other side to still air. In this paper symmetric laminated plates are considered, and there will be no coupling between bending and extension of a laminate. Therefore, the nondimensional governing equations can be expressed in terms of the nondimensional displacements  $\bar{w}$  (the deflection of the plate) and  $\bar{S}_x$  and  $\bar{S}_y$  (two rotations) for symmetric laminated plates. The resulting finite element equations are obtained using the shear deformable plate elements of Reddy<sup>4</sup>:

$$\begin{bmatrix} [K^{11}] & [K^{12}] & [K^{13}] \\ & [K^{22}] & [K^{23}] \\ \text{sym} & & [K^{33}] \end{bmatrix} \begin{Bmatrix} \{\bar{w}\} \\ \{\bar{S}_x\} \\ \{\bar{S}_y\} \end{Bmatrix} + a^4 \frac{\rho h}{E_2 h^3} \begin{Bmatrix} \{F\} \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} \{\ddot{\bar{w}}\} \\ \{\ddot{\bar{S}}_x\} \\ \{\ddot{\bar{S}}_y\} \end{Bmatrix} \quad (1)$$

where  $[K]$  and  $[M]$  are the stiffness and mass matrices, respectively;  $\{F\}$  are nodal forces due to the aerodynamic forces;  $a$ ,  $\rho$ , and  $h$  are the characteristic length, density, and thickness of the plate, respectively; and  $E_2$  is Young's modulus in the direction perpendicular to the fiber. Since the inertia terms of two rotations  $\bar{S}_x$  and  $\bar{S}_y$  become very small compared with the inertia terms of deflection  $\bar{w}$  for the thin panel, the finite element equations can be simplified through the Guyan reduction procedure. Thus, the number of equations to be solved is reduced and the finite element equation is only expressed in terms of  $\{\bar{w}\}$ .

By the application of the two-dimensional supersonic linearized theory to the panel flutter problem, we can obtain the nodal forces  $\{F\}$  as

$$\frac{a^4}{E_2 h^4} \{F\} = -\beta^* [A_f] \{\bar{w}\} - \bar{\mu} [A_d] \{\dot{\bar{w}}\} \quad (2)$$

where  $\beta^*$  is the nondimensional dynamic pressure parameter,  $[A_f]$  is the aerodynamic force matrix, and  $[A_d]$  is the aerodynamic damping matrix.

Assuming  $\bar{w} = \bar{w}^* e^{i\omega t}$ , we can obtain the following finite element equation:

$$[\bar{K}] + \beta^* [A_f] - k^* [M] \{\bar{w}^*\} = \{0\} \quad (3)$$

where  $[\bar{K}]$  is the condensed stiffness matrix obtained by the Guyan reduction,  $k^*$  are the nondimensional eigenvalues. As  $\beta^*$  increases monotonically from zero, two of these eigenvalues will approach each other and coalesce to  $k_{cr}^*$  at  $\beta^* = \beta_{cr}^*$  and become complex conjugate pairs for  $\beta^* > \beta_{cr}^*$ . The  $\beta_{cr}^*$  corresponds to the value of  $k^*$  at which the first coalescence occurs. Equation (3) can be solved directly by increasing values of  $\beta^*$  in many steps starting from zero. This procedure requires a tremendous amount of computational time. However,  $\{\bar{w}^*\}$  can be approximated by the linear combinations of a limited number of normal modes obtained from the free vibration analysis. Therefore, the flutter boundaries were evaluated using only a small number of normal modes in this analysis.

## Numerical Application

The following elastic properties of T300/5208 graphite/epoxy composite material are used:  $E_1 = 138 \times 10^5$  N/cm<sup>2</sup>,  $E_2 = E_3 = 9.7 \times 10^5$  N/cm<sup>2</sup>,  $G_{12} = G_{13} = 5.5 \times 10^5$  N/cm<sup>2</sup>,  $G_{23} = 4.1 \times 10^5$  N/cm<sup>2</sup>,  $\nu_{12} = 0.3$ , ply thickness  $h_p = 0.125$  mm, and  $\rho = 1.58 \times 10^{-5}$  N-s<sup>2</sup>/cm<sup>4</sup>. The convergence test of the solution has been performed for both the mesh size and the number of normal modes. There is a good convergence of solutions for a mesh size  $7 \times 7$  and 11 modes. The results obtained by using 11 modes agree well with those from Ref. 3.

Figure 1 gives the effect of the fiber angle for  $[(\pm\theta)_2]$ , rectangular laminates. It is shown that the flutter boundary ( $\beta_{cr}^*$ ) decreases dramatically as the fiber angle increases. Also,

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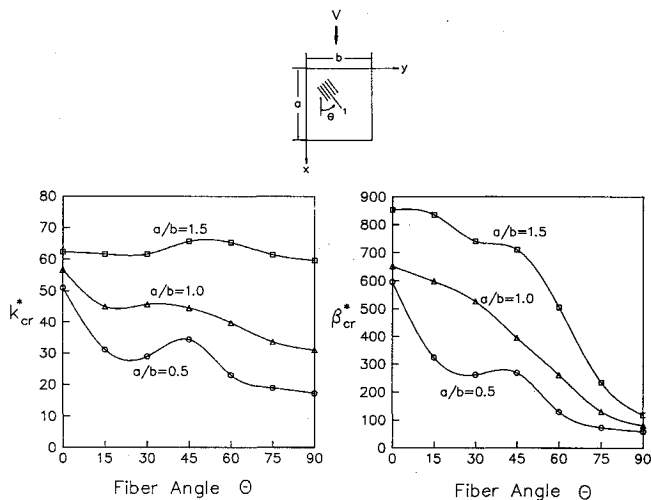


Fig. 1 Effect of fiber orientation on flutter boundaries for  $[(\pm\theta)_2]_s$  graphite/epoxy laminated rectangular panels ( $\phi = 0$ ).

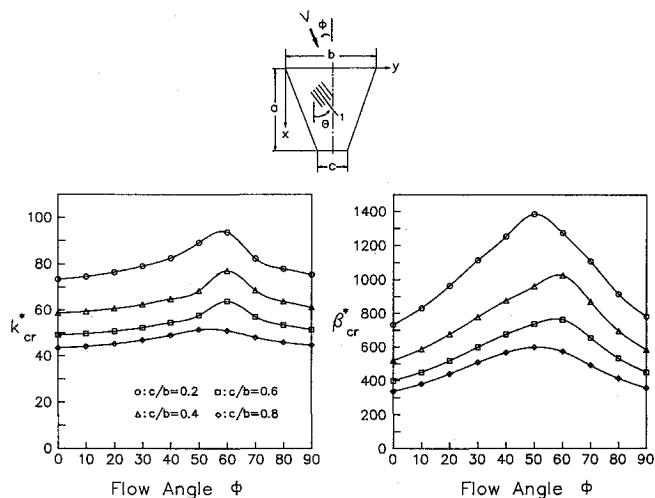


Fig. 3 Effect of flow direction on flutter boundaries for  $[45/\pm 45/0]_s$  laminated trapezoidal panels ( $a/b = 1.0$ ).

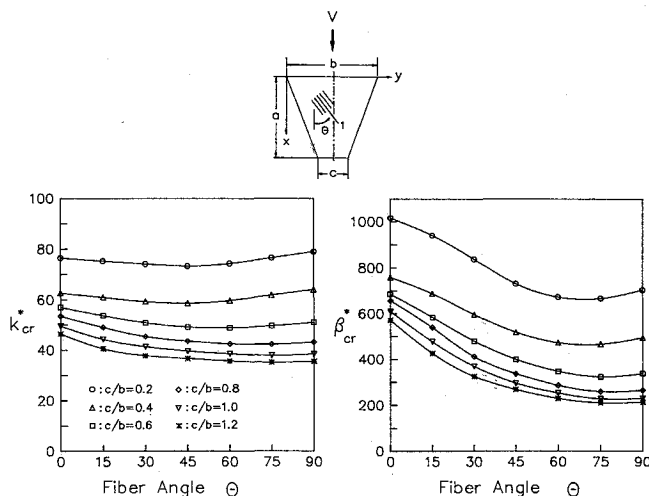


Fig. 2 Effect of fiber orientation on flutter boundaries for  $[\theta/\pm 45/0]_s$  graphite/epoxy composite trapezoidal panels ( $\phi = 0$ ,  $a/b = 1.0$ ).

as the aspect ratio ( $a/b$ ) increases, flutter boundaries increase. Figure 2 illustrates the effect of the fiber angle for  $[\theta/\pm 45/0]_s$  trapezoidal laminates. The flutter boundary ( $\beta_{cr}^*$ ) decreases gradually as the fiber angle increases. Also, flutter boundaries decrease as the lower to the upper side-length ratio ( $c/b$ ) increases. Figure 3 shows the effect of the flow direction ( $\phi$ ) for  $[45/\pm 45/0]_s$  laminates. The maximum value of flutter boundaries appears in the vicinity of  $\phi = \theta$ .

## Conclusions

The application of the finite element method based on the shear deformable theory to supersonic flutter problems has been presented. The results of the present method agree well with those in the available references. It is found that the influence of the panel geometry, its length ratio, the flow direction, and the fiber orientation greatly affect flutter boundaries of laminated plates with clamped edges. The maximum value of the flutter dynamic pressure appears in the vicinity of  $\phi = \theta$  (i.e., when the flow angle is consistent with the fiber angle) for both  $[(\pm\theta)_2]_s$  rectangular and  $[\theta/\pm 45/0]_s$  trapezoidal plates.

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